## Exercise 25

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$y\sin 2x = x\cos 2y, \quad (\pi/2, \pi/4)$$

## Solution

The aim is to evaluate y' at  $x = \pi/2$  and  $y = \pi/4$  in order to find the slope there. Differentiate both sides of the given equation with respect to x.

$$\frac{d}{dx}(y\sin 2x) = \frac{d}{dx}(x\cos 2y)$$
$$\left[\frac{d}{dx}(y)\right]\sin 2x + y\left[\frac{d}{dx}(\sin 2x)\right] = \left[\frac{d}{dx}(x)\right]\cos 2y + x\left[\frac{d}{dx}(\cos 2y)\right]$$
$$(y')\sin 2x + y\left[(\cos 2x) \cdot \frac{d}{dx}(2x)\right] = (1)\cos 2y + x\left[(-\sin 2y) \cdot \frac{d}{dx}(2y)\right]$$
$$y'\sin 2x + y\left[(\cos 2x) \cdot (2)\right] = \cos 2y + x\left[(-\sin 2y) \cdot (2y')\right]$$
$$y'\sin 2x + 2y\cos 2x = \cos 2y - 2xy'\sin 2y$$

Solve for y'.

$$(\sin 2x + 2x \sin 2y)y' = \cos 2y - 2y \cos 2x$$
$$y' = \frac{\cos 2y - 2y \cos 2x}{\sin 2x + 2x \sin 2y}$$

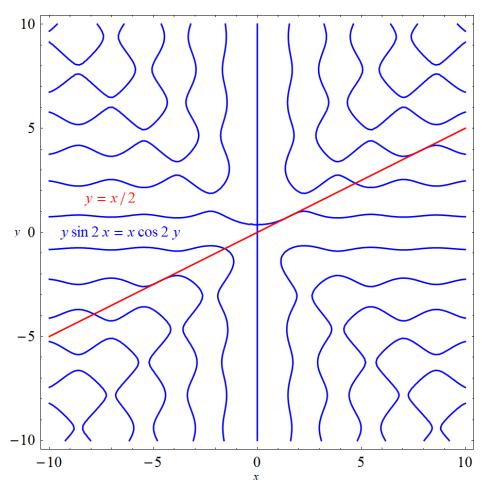
Evaluate y' at  $x = \pi/2$  and  $y = \pi/4$ .

$$y'\left(\frac{\pi}{2},\frac{\pi}{4}\right) = \frac{\cos\left[2\left(\frac{\pi}{4}\right)\right] - 2\left(\frac{\pi}{4}\right)\cos\left[2\left(\frac{\pi}{2}\right)\right]}{\sin\left[2\left(\frac{\pi}{2}\right)\right] + 2\left(\frac{\pi}{2}\right)\sin\left[2\left(\frac{\pi}{4}\right)\right]} = \frac{1}{2}$$

Therefore, the equation of the tangent line to the curve represented by  $y \sin 2x = x \cos 2y$  at  $(\pi/2, \pi/4)$  is

$$y - \frac{\pi}{4} = \frac{1}{2} \left( x - \frac{\pi}{2} \right),$$
$$y = \frac{x}{2}.$$

or



Below is a graph of the curve and the tangent line at  $(\pi/2, \pi/4)$ .